

N. VILENKIN

COMBINATORIAL MATHEMATICS

FOR RECREATION



$$P_n = n!$$

00, 01, 02, 03
10, 11, 12, 13
20, 21, 22, 23
30, 31, 32, 33



$n=5$

MIR PUBLISHERS, MOSCOW

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Translated from the Russian by George Yankovsky

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MOSCOW

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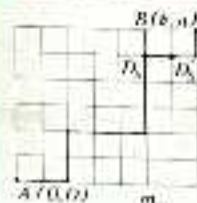
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"A science club consists of 25 members. The task at hand is to elect a president, a vice president, scientific secretary and treasurer. In how many ways can this choice be made if each member of the club can occupy only one post at a time?"

You will find the answers to this question and many others in this book.

$$A_n^k = \frac{n!}{(n-k)!}$$

$$C_n^k = \frac{n!}{(n-k)! k!}$$



$$1$$

$$1 \quad 1$$

$$1 \quad 2 \quad 1$$

$$1 \quad 3 \quad 3 \quad 1$$

$$1 \quad 4 \quad 6 \quad 4 \quad 1$$